Final Review 1

Basic Functions

1) Give the definition of a function and give two examples.

2) Find the linear function $f(x)$ if $f(3) = 5$ and $f(-1) = 7$. Write the equation for $f(x)$ in both point-slope and slope-intercept form.

3) Find the equations for the lines parallel to and perpendicular to the line $4x + 3y - 6 = 0$ and passing through the point (3, -1).

4) Find the x-intercepts (if they exist) and the vertex algebraically and graph each function by hand.

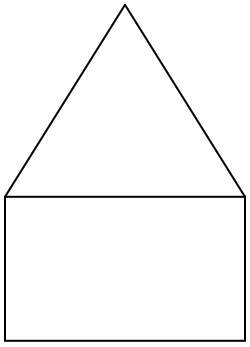
a) $f(x) = x^2 - 2x - 8$ b) $f(x) = x^2 + 2x + 5$

5) A special window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 16 feet, what dimensions will admit the most light (maximize the area)?

a) Express the area of the window as a function of the width, x, of the rectangle.

b) Include a meaningful graph for the problem.

c) Find the dimensions that maximize the area.



Domains, Compositions, and Inverses

6) Find the domain of each function algebraically and support your response graphically.

a) $f(x) = \sqrt{4-5x}$

b) $f(x) = \dfrac{x-4}{x^2+5x-36}$

7) If $f(x) = 3x^2 + 1$ and $g(x) = \sqrt{3x-2}$, find the following and simplify:

a) Evaluate $f(-1)$ and $g(2) - 2$ and $f(x + 2)$

b) Find the domains of $f(x)$ and $g(x)$ algebraically and support graphically.

c) Determine whether $f(x)$ and $g(x)$ have an inverse function. If so, find $f^{-1}(x)$ and $g^{-1}(x)$.

d) Find $(f - g)(3)$ and $(f + g)(-2)$

e) Find $(f \circ g)(x)$ and $(g \circ f)(x)$, simplify, and find their domains algebraically.

Graphing

8) Sketch the graph of each polynomial by hand. Use what you know about the degree, leading coefficient, multiple zeros, intercepts, and plotting sufficient points. Check your graph with a graphing calculator.
   a) $f(x) = (x + 2)(x)^3$
   b) $f(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$

9) Sketch a graph of each rational function by hand. Show evidence of your understanding of intercepts and vertical, horizontal, and slant asymptotes. Check yourself with a graphing calculator.

   a) $f(x) = \dfrac{3x}{x^2 - x - 2}$

   b) $f(x) = \dfrac{x^3}{2x^2 - 8}$

10) Graph the function $f(x) = -x^4 + 2x^3 + 4x^2 - 3$ on the interval (-5, 5). Approximate any relative maxima and minima rounded to two decimal places. Determine the interval(s) where the function is increasing and the interval(s) where it is decreasing to two decimal places.

Higher Degree Polynomials and Complex Numbers

11) Use all you know to find the zeros of the polynomials below. Write the polynomial in factored form, and sketch a graph by hand. Show evidence of your understanding of Descarte's Rule of Signs, the Rational Zero Test, upper and lower bounds, and synthetic division. Check yourself with a graphing Calculator
   a) $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$
   b) $f(x) = x^4 + 2x^3 + 10x^2 + 18x + 9$

12) Perform the operations and write the result in standard form.
   a) $(11 - 2i) + (-3 + 6i)$
   b) $(6 - 2i)(2 - 3i)$
   c) $\dfrac{8 - 7i}{1 - 2i}$

13) Identify the common function $f(x)$ and describe the sequence of transformations from f to h. Make a sketch of $h(x)$ for each problem.
   a) $h(x) = -2(x + 1)^3 + 3$
   b) $h(x) = |x - 4| + 2$

Answers to Pre-Calculus Review 1

1) A function has one *y* for every *x* (see page 16 for the formal definition), you can use the vertical line test to determine if there exists a functional relationship.

2) point-slope form: $y = -\frac{1}{2}(x - 3) + 5$ or $y = -\frac{1}{2}(x + 1) + 7$
Slope-intercept: $y = -\frac{1}{2}x + 6.5$

3) Parallel: $y = -4/3(x - 3) - 1$ or $y = -4/3x + 3$
Perpendicular $y = \frac{3}{4}(x - 3) - 1$ or $y = \frac{3}{4}x - 3.25$

4) a) $(x+2)(x-4)$, vertex at $(1, -9)$
b) no real roots, vertex at $(-1,4)$

5) $x = 3.749$ $y = 2.37669$ area = 29.99

6) a) $x \leq 0.8$
b) $x \neq -9, 4$

7) a) $f(-1) = 4$ $g(2) - 2 = 0$ $f(x+2) = 3x^2 + 12x + 13$
b) f(x) all reals $g(x)$ $x \geq 2/3$
c) f(x) no inverse g(x) has an inverse $= (x^2 + 2)/3$
d) $(f - g)(3) \approx 25.35$ $(f + g)(-2)$ = no solution
e) $(f \circ g)(x) = 9x - 5$, domain $x \geq 2/3$ $(g \circ f)(x) = \sqrt{(9x^2 + 1)}$, domain all reals

8) Check on calculator

9) a) Check graph on calculator, vertical asymptotes at $x = 2, -1$
b) Check graph on calculator, vertical asymptotes at $x = 2, -2$; slant asymptote at $y = \frac{1}{2}x$

10) Relative maxima at (-0.85, -1.86) and (2.35, 14.55)
Relative minimum (0, -3)
Increasing over the intervals $(-\infty, -0.85)$ and $(0, 2.35)$
Decreasing over the intervals $(-0.85, 0)$ and $(2.35, \infty)$

11) a) $f(x) = (x - 1)(x - 1)(x + 2)(x - 2i)(x + 2i)$ $x = 1, 1, -2, 2i, -2i$
b) $f(x) = (x + 1)(x + 1)(x - 3i)(x + 3i)$ $x = -1, -1, 3i, -3i$

12) a) $8 + 4i$
b) $6 - 22i$
c) $22/5 + 9/5i$

13) a)Cubic function; stretched vertically by 2, reflected over the x-axis, shifted left 1and up 3
b) Absolute value function; shifted right by 4 and up by 2